POWER-LAW PRODUCTION FUNCTIONS OF COMPLEX VARIABLES

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Annotation: In statistics, a **power law** is a functional relationship between two quantities, where a relative change in one quantity results in a proportional relative change in the other quantity, independent of the initial size of those quantities: one quantity varies as a power of another. For instance, considering the area of a square in terms of the length of its side, if the length is doubled, the area is multiplied by a factor of four.

Key words: Distributions of a wide, sizes of activity patterns of neuronal populations, frequencies of words in most languages

The distributions of a wide variety of physical, biological, and man-made phenomena approximately follow a power law over a wide range of magnitudes: these include the sizes of craters on the moon and of solar flares, the foraging pattern of various species, the sizes of activity patterns of neuronal populations, the frequencies of words in most languages, frequencies of family names, the species richness in clades of organisms, the sizes of power outages, criminal charges per convict, volcanic eruptions, human judgments of stimulus intensity and many other quantities. Few empirical distributions fit a power law for all their values, but rather follow a power law in the tail. Acoustic attenuation follows frequency power-laws within wide frequency bands for many complex media or relationships between biological variables are among the best known power-law functions in nature.

Although more sophisticated and robust methods have been proposed, the most frequently used graphical methods of identifying power-law probability distributions using random samples are Pareto quantile-quantile plots (or Pareto Q–Q plots),[*citation needed*] mean residual life plots and log–log plots. Another, more robust graphical method uses bundles of residual quantile functions. (Please keep in mind that power-law distributions are also called Pareto-type distributions.) It is assumed here that a random sample is obtained from a probability distribution, and that we want to know if the tail of the distribution follows a power law (in other words, we want to know if the distribution has a "Pareto tail"). Here, the random sample is called "the data".

Pareto Q–Q plots compare the quantiles of the log-transformed data to the corresponding quantiles of an exponential distribution with mean 1 (or to the quantiles of a standard Pareto distribution) by plotting the former versus the latter. If the resultant scatterplot suggests that the plotted points "asymptotically converge" to a straight line, then a power-law distribution should be suspected. A limitation of Pareto Q–Q plots is that they behave poorly when the tail index (also called Pareto index) is close to 0, because Pareto Q–Q plots are not designed to identify distributions with slowly varying tails.

A straight line on a log–log plot is necessary but insufficient evidence for power-laws, the slope of the straight line corresponds to the power law exponent.

Log–log plots are an alternative way of graphically examining the tail of a distribution using a random sample. Caution has to be exercised however as a log–log plot is necessary but insufficient evidence for a power law relationship, as many non power-law distributions will appear as straight lines on a log–log plot. This method consists of plotting the logarithm of an estimator of the probability that a particular number of the distribution occurs versus the logarithm of that particular number. Usually, this estimator is the proportion of times that the number occurs in the data set. If the points in the plot tend to "converge" to a straight line for large numbers in the x axis, then the researcher concludes that the distribution has a power-law tail. Examples of the application of these types of plot have been published. A disadvantage of these plots is that, in order for them to provide reliable results, they require huge amounts of data. In addition, they are appropriate only for discrete (or grouped) data

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