SOME PROPERTIES OF COVARIANT FUNCTORS ON THE CATEGORY *Comp*

T. F. Zhuraev
K.R.Zhuvonov,
G.J.Eshmirzaeva
e-mail: Tursunzhuraev@mail.ru, qamariddin.j@mail.ru

In this note, we consider covariant functors in the categories of $Comp-$compact spaces, Metr-metrizable spaces, S-stratifiable spaces, $\aleph-$spaces, paracompact p-spaces, and continuous self-maps. It is proved that functors with finite supports acting in certain categories preserve finite-dimensional spaces and weakly countable spaces. Closed functors with finite support are defined and it is proved that closed functors preserve the class of S-spaces.

Recall the definition and some normality properties of a covariant functor $F: Comp \to Comp$ acting in the category of compact sets. The functor F is said to be:

Stores the empty set and point if $F(\varnothing)=\varnothing$ and $F(\{1\})=\{1\}$ where $\{k\}, k\geq 0$ we denote the set of non-negative integers - $\{0,1,...,k-1\}$ less than k. In this terminology $\{0\}=\varnothing$;

Monomorphic if for every (topological) embedding. $f: A \to X$ the mapping $F(f): F(A) \to F(X)$ is an embedding.

Epimorphic if, for every mapping $f: A \to Y$ onto Y, the mapping $F(f): F(A) \to F(Y)$ is also a mapping "to"";

Preserves intersections if for any family $\{A_\alpha : \alpha \in A\}$ of closed subsets of X and identical embeddings $i_\alpha : A_\alpha \to X$, mapping $F(i_\alpha): \bigcap\{F(A_\alpha): \alpha \in A\} \to X$ defined by $F(i)(\alpha) = F(i_A)(\alpha)$, is an embedding for every $\alpha \in A$;

Pre-images if for every mapping $f: X \to Y$ and every closed set $A \subset Y$ the mapping $F(f|_{f^{-1}(A)})(f^{-1}(A)) \to F(A)$ is a homeomorphism;

Preserves weight if $\omega(F(X)) = \omega(X)$ for an infinite compact space X;

Continuous if for every inverse spectrum $S=\{X_\alpha; \pi_\beta^\alpha : \alpha \in A\}$ from bicompacta, a homeomorphism is a mapping $f: F(\lim S) \to \lim F(S)$ which is the limit of mappings $F(\pi_\alpha)$ if $\pi_\alpha : \lim S \to X_\alpha$-through projections of the S spectrum.

In what follows, we assume that all functors under consideration are monomorphic and preserve intersections. We also assume that all functors preserve non-empty spaces. This restriction is not essential, since by doing so we exclude from consideration only the empty functor, i.e., the functor F that maps any space to the empty set. Indeed, let $F(X)=\varnothing$ for some non-empty bicompact set X.

Then $F(X)=F(1)=\varnothing$ since F is monomorphic. Now let Y be an arbitrary non-empty compact set. Consider a constant mapping $f:Y\to 1$ Then $F(f)(F(f))\subset F(1)=\varnothing$. Hence the space $F(Y)$ is empty because it maps to the empty set. So, we have proved that there is a unique monomorphic functor that preserves non-empty sets.

Let $F:Comp\to Comp$ be a functor. $C(X,Y)$ denotes the space of continuous mappings from X and Y in the compact-open topology. In particular, $C(\{k\},Y)$ is naturally homeomorphic to the k th power of Y^k of the space Y.

The map $\xi:\{k\}\to Y$ is assigned a dot $(\xi(0),...\xi(k-1))\in Y^k$.

For a functor F, a bicompact space X of a natural number k, we define a mapping $\pi_{F,X,k}:C(\{k\},X)\times F(\{k\})\to F(X)$ by $\pi_{F,X,k}(\xi,a)=F(\xi)(a)$ where $\xi\in C(\{k\},X),\alpha\in F(\{k\})$.

When it is clear which functor and bicompact space we are talking about, we will denote the mapping $\pi_{F,X,k}$ by $\pi_{X,k}$ or π_k.

Necessary facts related to covariant functors and their properties can be found in [1-2].

Lemma 1. Let $F:Tych\to Tych$ be monomorphic, preserving intersections, inverse images of mappings, by continuous supports of a functor of degree $\le n$. Then for any $i=\overline{0,n}$ the set $F_{i-i}(i)$ is open -closed in $F(i)$ if $F(Comp)\subset Comp$.

If the functor satisfies the conditions of Lemma 1, then by Theorem 5.1 [3] we have

Theorem 1. If the functor F satisfies the conditions of Lemma 1, then any Tychonoff space X and any $i=\overline{0,n}$ map $\pi_{F,X,i}:X^i\times F(i)\to F_i(X)$ factorial.

Theorem 2. Let $F:Tych\to Tych$ be a monomorphic, intersection-preserving, preimage-preserving functor of degree $\le n$, a set $F_{n-1}(\tilde{n})$ is open in $F(\tilde{n})$, then the functor F is continuously supported if the mapping

$\pi_{FXi}:X^n\times F(\tilde{n})\to F_n(X)$ closed

Definition. A continuous functor F $Tych\to Tych$ with finite supports $\le n$ is called closed if the map $\pi_{Fxn}:X^n\times F(\tilde{n})\to F(X)$ is closed.

A T_1-space X is called stratifiable [4] (lace, short S-space) if each open set $U\subset X$ can be associated with the sequence $\{U_n:n\in N\}$ open subsets in such a way that the following conditions are satisfied;

a) $\overline{U_n}\subset U$ for all $n\in N$; b) $\cup\{U_n:n\in N\}=U$;

c) if $U\subset V$, then $U_n\subset V_n$ for all n.

Note[5] that S-spaces are perfectly normal and paracompact, and also a finite union and a countable product of an S-space is again a S-space. It was shown in [5] that every S-space is a σ-space. Hence S-space is a paracompact σ-space.

If the functor $F: Tych \to Tych$ is closed, then the space $F(X)$ is an S- space if and only if X is an S-space and $F(\tilde{n})$ is also S-space. Since the space $X^n \times F(\tilde{n})$ is S- the space $[4]$. S-space is preserved under closed mappings $[3]$.

Therefore, it takes place.

Theorem 3. Normal closed functors $F: Tych \to Tych$ with finite supports $\leq n$ preserve the category of S-spaces.

Definition [6]. A Hausdorff space X is called an $\aleph$-space if it can be mapped onto some S-space S by a perfect mapping.

Let $F: Tych \to Tych$ be a normal or seminormal functor preserving S-spaces and perfect mappings, i.e. $F(St) \subset S$ and $F(f)$ is a perfect mapping if f is a perfect mapping.

In this case, there is

Theorem 4. Let $F: Tych \to Tych$ be a seminormal functor preserving S- spaces and perfect mappings. Then the functor F preserves $\aleph-$spaces.

Since closed functors preserve the category of S-spaces and perfect mappings, we therefore have

Theorem 5. Closed functors $F: S \to S$ preserve $\aleph$-spaces.

For a seminormal functor $F: Comp \to Comp$ and a Tikhonov space X we set $F_\beta(X) = \{a \in F(\beta X): supp a \subset X\}$, where βX –Stono is the Chekhov extension of the space X.

Reference

1. Fedorchuk V.V., Filippov V.V. General topology. Basic structures. M. From Moscow State University, 1988.p.252.

2. Shchepin E.V. Functors and uncountable powers of compact sets. UMN, 1981, V.36, 3, S.3-62.

3. Basmanov V.N. Covariant functors of finite degrees on the category of compact spaces. Foundation. and applied mat.M.1996, T.2.issue 3.p.637-654.

4. Borges C.J.R. On stratifiable spaces Racif.J.Math., 1966, V.17, 1,pp 1-16.

5. Heath R.W., Hadel R.E. Characterizations of σ-spaces. Fund.Math. v.77, 3, 1973, pp.271-275.

6. Mardeshich S., Shostak A. On perfect inverse images of lacy spaces. Uspekhi Mat. Nauk, 1986, T-35, issue 3, pp. 84-93.