MATHEMATICAL ANALYSIS WITH SYSTEMS "MATHEMATICS" AND WEB MATH

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Abstract
We consider teaching the Continuous functions topic of Calculus using Mathematica and Web Mathematica. We show that it makes possible not only to visualize the traditional approach of teaching but helps the students to reach the deeper understanding of continuity. It is achieved by the best motivation and explication of mathematical concepts.

Keywords: continuity, continuous function, function graph, Mathematica.

Use of Computer Algebras, or CAS in English, in Universities USA and Western Europe for educational purposes began almost simultaneously with their appearance in the world in the 70s-80s of the last century. Author in University of Maryland in 1992 discovered in a computer network University of the system "Mathematics", "Maple", Reduce and several highly specialized programs for symbolic algebraic calculations. However, in bookstores in the United States, the main place at that time was occupied by textbooks on CAS, and not by their applications for the study of academic disciplines. But already in 1995 the situation has changed significantly, and many textbooks on mathematical analysis and linear algebra. Currently leadership in use in education is held by such computer algebras like "Mathematics", "Maple" and "Matkad", which are integrated systems of symbolic, graphic and numerical mathematical calculations. ABOUT the degree of implementation of these systems in education can be judged by the fact that among those issued in 2007 by the largest publishing house of scientific and educational literature CRC Press (USA) of math textbooks 25% used in his presentation of computer algebras.
The largest share among textbooks using CAS is occupied by manuals on the discipline Calculus (Calculus). This circumstance can be explained both by the importance of the discipline for applications and, consequently, by its inclusion in the curricula of almost all engineering, natural science and economic specialties. There are two areas in which the influence of CAS on the teaching of calculus is greatest. This is the visualization of mathematical objects: sequences, functions, curves and surfaces, discrete data, etc., as well as the automation of calculations that make it possible to include in the curricula the

implementation of projects that are interesting in themselves for students and are of applied importance for physics, mechanics, medicine , economics, ecology, etc.
Meanwhile, the potential influence of CAS on the discipline of mathematical analysis, in our opinion, can be much deeper and affect the explication of the basic concepts of the discipline: a real number, a continuous function, a derivative, an integral. In this paper, we will discuss an important, albeit private, issue related to the use of computer algebras to study the continuity of functions and their graphs. We will confine ourselves to consideration of the "Mathematics" system [1].
1. Continuity, continuous functions
Authors and reviewers of the most popular and reprinted university textbooks on mathematical analysis as the main
The principle underlying the presentation of the material is called mathematical rigor and logical harmony. So Academician of the Academy of Sciences of the USSR A.N. Kolmogorov in
In his review of the textbook [2], he writes: “The complete rigor of presentation ... is connected with accessibility and completeness, as well as with the cultivation of the habit of dealing with real problems of natural science.” We see that in the first place "complete rigor of presentation."
The authors of the textbook [3] in their preface, in particular, write: "The authors tried to avoid excessive detail, but not to the detriment of logical rigor." In the preface to the textbook [4], we meet with the following position: "... we have completely abandoned the concept of the limit of a variable ... This allows us to make the presentation logically more transparent."
However, in our opinion, it is precisely with logical transparency and harmony in the textbooks of mathematical analysis of the pre-computer era that far from everything is in order. Indeed, in some of them, real numbers, one of the main and most important objects of mathematical analysis, are introduced according to Dedekind with the help of the concept of "sections" in the set of rational numbers. But in the chapters following the real numbers, the "section" object is never mentioned. As a rule, everywhere below, instead of the term number as a function argument, the term “point” begins to be used: “consider the value of the function f (x) at the point 0.1”.
The sections were needed only in order to prove the existence of a limit of a monotonically increasing sequence or to justify any its equivalent statement. It is on these assertions that the entire edifice of analysis is built, and, in particular, the properties of continuous functions are studied.
The situation is similar for two other ways of introducing real numbers: according to Weierstrass using formal infinite decimal fractions or according to Cantor using the fundamental Cauchy sequences of rational numbers. Everywhere the continuity of functions is defined with the help of limits. In this case, the logically preceding concept of the continuity of the real numerical axis is not used.

In addition to the fascination with logical rigor, the existing textbooks of mathematical analysis can be reproached for the lack of motivation for the introduced mathematical terms, although it is impossible to understand any definition without a preliminary explanation of the need for its introduction. For example, why are functions that satisfy the definition: “a function f (x) is called continuous at a point a if for any ε there is such a δ that the inequality | | f (x) – f (a) |< ε is executed whenever | x – a |< δ »» - are called continuous? Is it possible, without prejudice to understanding the definition, to replace the word "continuous" with the word "pleasant", or "pleasant in all respects"?

Neglect of motivations or presentation of leading considerations has long historical roots in mathematics. Descartes also stated: “We should not experimentally verify the initial positions of our theories: these are just arbitrary axioms and their relation to reality has nothing to do with science. It is equally meaningless to compare with reality the final conclusions: they are unlikely to agree with it better than the original axioms. What is really important is according to strict rules of logic transform axioms into final results, avoiding any involvement of the imagination. To make geometry a science, it is necessary to banish from her drawings are traces of experiments, on the one hand, and food for imagination on the other. (Quoted from [5], p. 249).

Returning to the above definition of a continuous function, note that the meaning of the term continuous function is presented to the authors textbooks are so intuitive that it is not worth wasting time on it clarification. It is associated, apparently, with a visual image of the graph continuous function as a continuous line without breaks. Useful for this note that the concept of continuity was defined above at a point, and not on a segment or interval. Therefore, the image of a line without breaks is inadequate. Indeed, it is easy give examples of functions that are continuous only at one point and have discontinuities at all other points in its domain of definition. For example, the function φ(x) = xδ (x) , где δ (x) , – Dirichlet function equal to +1 for irrational x And equal to -1 for irrational x is continuous only at a point 0. In all the rest points of the real real axis it has discontinuities of the second kind, i.e. does not have no limits on the left or right.

Function Graph φ(x) can be represented as bisectors of the first and third and second and fourth quadrants, i.e. it consists of two solid lines. This shows the irrelevance of the appeal to visual geometric images.

2. Graphs of continuous functions with the system "Mathematics"

In the textbooks of the pre-computer era, there is no logical harmony and when covering the topic Graphs of continuous functions. Indeed, on the one hand the following definition of a graph of a function is given: “a graph of a function f(x) is the set of all ordered pairs (x, f(x))". Those. chart is a set that is naturally identified with a set of points M(x, f(x)) real coordinate plane. Hence naturally the conclusion suggests itself: the graph of the function needs to be drawn, marking points on paper, those.

Meanwhile, the graph is tacitly identified with some line on a plane that is drawn without lifting the pencil from the paper. In the same time, the definition of a line (curve) in mathematical analysis uses, in the end account, the concept of a graph of a function. But if the graph is identified with the line, and the graph continuous function to identify with a continuous line, it is not clear why math professors spend time proving the following obvious statements.

Theorem. Let the function be continuous on the interval [a, b] and on its ends takes values of different signs. Then there is at least one point on the segment in which the function takes a zero value, i.e. crosses the axis abscissa.

Indeed, like a line that can be drawn on a plane without tearing pencil from paper, may not cross the x-axis, moving from the top half-plane to the bottom or vice versa from the bottom to the top? The best way to get rid of this confusion: study the topic of graphics functions using the "Mathematics" system. In all computer algebras there are means for visualization of discrete data. In "Mathematics" command ListPlot[data], where data is a finite set of ordered pairs of the form {a,b}. The result of the command execution is a set of points on the coordinate planes. If b = f (a), then data consists of a set of points (not all) of the graph functions f(x) .

Consider an example of plotting a graph. Let f (x) = sin(x). We implement the following way of constructing some finite set of points of the graph of this trigonometric function. We will plot the graph points using trigonometric identities. First, these are the half angle formulas:

$$\sin(x)=\sqrt{\frac{1-\cos(2x)}{2}},\qquad \cos(x)=\sqrt{\frac{1+\cos(2x)}{2}} \qquad (1)$$

Second, the formulas for the sine and cosine of the sum of two angles:

$\sin(x + y) = \sin(x)\cos(y) + \cos(x)\sin(y)$ $\cos(x + y) = \cos(x)\cos(y) - \sin(x)\sin(y)$ (2)

Let's build a scatter plot of the function sin (x) with a step of π/96. Let's make an important remark. The "Mathematics" system can calculate the values of trigonometric functions at any point with any predetermined accuracy and perfectly draws a sine graph. Therefore, to emphasize that our calculations will be completely independent of the knowledge of trigonometric functions, which are incorporated into the "Mathematics" system, instead of functions with the names sin (x) and cos (x), we will create our own functions, which we will denote by Mysin (x) and My cos (x) .

First step: we enter formulas (1) into the computer in the syntax of "Mathematics" and take into account that from geometry we know the values of the sine and cosine functions for the angle π / 6:

$$My\sin[x\_]:=\sqrt{\frac{1-My\cos[x]}{2}};\ My\sin[\pi/6]=1/2;$$
$$My\cos[x\_]:=\sqrt{\frac{1+My\cos[x]}{2}};\ My\cos[\pi/6]=\sqrt{3}/2 \qquad (3)$$

After these formulas are calculated, i.e. relevant definitions entered into the computer "Mathematics", it will be possible to calculate sine and cosine functions for any view angles π /(6∗ 2^ n) In particular, when n = 4 get the required angle π / 96. Calculation mechanism: the so-called recursive programming. For example, to calculate mysin [π / 96] computer "Mathematics" in accordance with the information entered into it definitions (3) will turn to the angle [π / 96], And mysin [π / 96] will be calculated from result $\sqrt{\frac{1-\sqrt{3}}{2}}$.

Now we enter formulas (2) into the computer for y = π / 96 and received approximate values Mysin[π / 96], My cos[π / 96].

Clear[Mysin,My cos] Mysin[x]:= Mysin[x –π /96]My cos[π /96]+ Mysin[π /96]Mycos[x –π /96] My cos[x _]:= My cos[x –π /96]My cos[π /96]– Mysin[x –π /96]Mysin[π /96]These definitions allow us to recursively calculate the values of our functions in points away from the point [π / 96].at distances multiples [π / 96]. For example, Mysin[π / 6 +π /96] = 0.528072. Now we can draw the points of the graph of the function mysin[x] on the segment [0,2π]. To do this, we calculate a set of 192 pairs of graph points, i.e. numbers of the form {x,mysin[x]}:

data = Table[{x,Mysin[x]},{x,π /96,2π,π /96}];

Using the ListPlot command, we draw these points on the plane (Fig. 1).

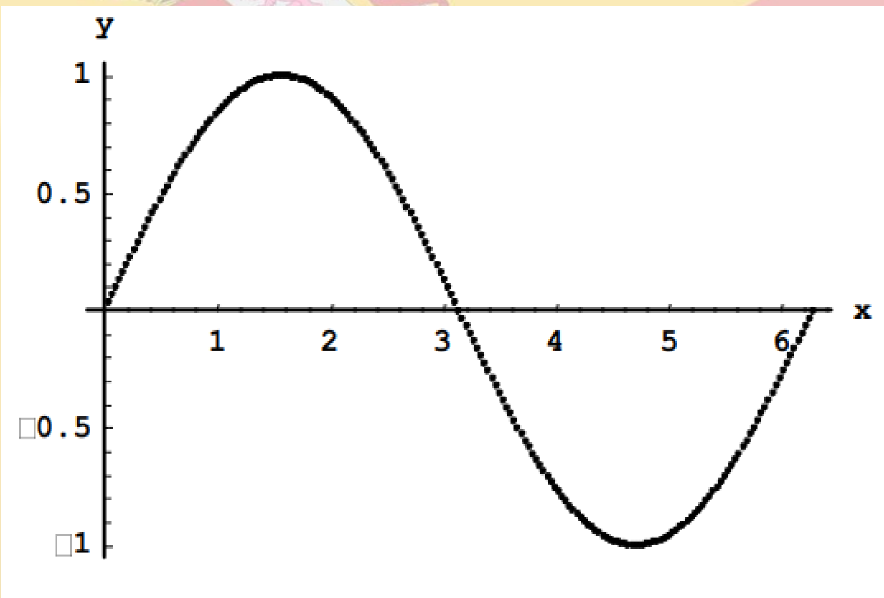

Fig.1. Graph of the function Mysin[x] with a step of π/96.

In the presented figure, the pointwise nature of the graph is clearly visible. In addition, it can be seen that some of the points of the graph, overlapping, merge into a solid line, which is perceived by the graph of any function when it is traditionally drawn with a pencil and paper. If we decrease the graph step, i.e. go to step π /192 or even less, then all discrete points of the graph will merge into one solid line - the function graph found everywhere in mathematical and educational literature sin(x).

3. Investigation of the continuity of functions using the system "Mathematics" Above we gave the definition of continuous at a fixed point functions "in the language ε –δ", or the Cauchy definition. Other commonly used definition - a definition in the language of sequences, or a definition by Heine: "function f(x) continuous at point a, if for any sequence, converging to a, the sequence of function values converges to f(a)". Except Moreover, with the help of the concepts of the left and right limits of a function at a point, classification of break points.

If we use Heine's definition of continuity, then with the help of "Mathematicians" can be attributed to the study of the continuity of a function at a point geometric visibility. To do this, as in the previous section, we will study the behavior of discrete points of a function graph.

Note that Heine's definition means that in the case of continuity, the set of points of the graph of a function converges to the point (a, f (a)) of the graph. Conversely, the convergence of all such sequences to the point (a, f (a)) implies the continuity of the function at the point.

Consider as an example the function f (x) = (1– cos(x))/ x^2. This the function is not defined by the formula at the point x = 0, i.e. has a discontinuity at this point. Task: what is the nature of this gap, i.e. whether it is disposable, first or second kind.

In order to get a first impression of the behavior of the function at the point x = 0, consider a sequence with elements (–1)^ n / n. Compute the set data = Table[{(–1)^n/ n, f[(–1)^n/ n]},{n,5,50}]; containing forty-five elements, and use the ListPlot command to present it as points on a plane (Fig. 2).

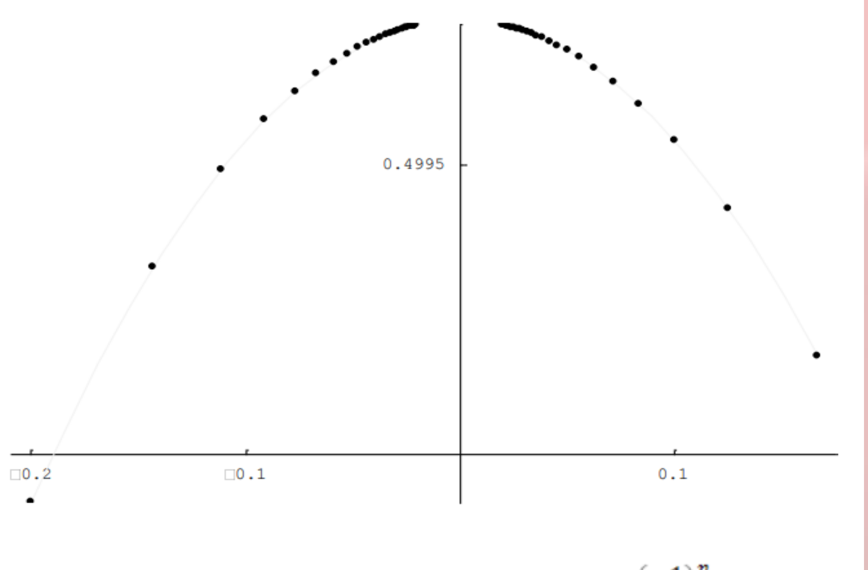

Рис2. Множество точек графика для последовательности $(-1)^n/n$

Studying this figure, we can make an assumption that the sequence of points of the graph converges both from the left and from the right to the point (0,0.5). This conclusion will become even more convincing if you make a cartoon, the frames of which are drawings similar to the one made, but the number of points on which increases by one from frame to frame. Then the aspiration of the graph points to the point (0,0.5) will be more convincing. In Fig. 2, the points of the graph, condensing, do not reach the limit point. You can do the following: either increase the number of elements of the sequence, or, with the same number of elements, choose a sequence that converges faster to point 0. Say, you can take the sequence (–1)^n / n^2, then the points of the graph will close around the limit (0.0.5).

In conclusion of this section, we emphasize that a visual study of the behavior of the graph points only allows us to hypothesize that the function under consideration has a removable discontinuity at point 0, but does not prove this fact. After the hypothesis is stated, it should be proved (or refuted) by standard mathematical methods. In this case, calculate the limit

lim_(x– > 0)(1– cos(x)/ x^2) = lim_(x– > 0)sin(x^2)/ x^2 =1/ 2

4. Continuity with WebMath

Teaching mathematical disciplines with the help of "Mathematics", with all its undeniable advantages, has a significant drawback that, along with the high cost of this software product, prevents it from

widespread use in higher education. The teacher and students need, at least superficially, to know the command system and the syntax of the input mathematical expressions. This shortcoming is eliminated if instead of "Mathematics" we use a remote settlement system via the Internet, called Webmathematics.

WebMathematics is a web interface for the integrated system of symbolic, graphical and numerical calculations "Mathematics". When teaching mathematical disciplines with the help of WebMathematics, electronic teaching aids are used, equipped with programs for performing symbolic, graphical or numerical calculations.

The WebMathematics system is an application program for the Java server that allows you to carry out symbolic, graphical and numerical calculations over the Internet based on specially written scientific and educational interactive electronic documents hosted on the server. To carry out calculations, the user only needs to have a standard web browser.

WebMathematica creates on the server and sends to the user, upon request, special web pages that contain HTML forms with fields for entering user information. After the information is entered and the user presses the "Calculate" button, the page is sent back to the server, the commands are executed, and the results of the calculations are pasted into a new page sent to the user.

It should be noted that with the help of WebMathematics it is impossible to get direct access to the Mathematics computer. You can execute only those commands that are contained in the body of HTML forms and the data for which the user types into the input fields of these forms. Thus, only a relatively small set of commands is actually available to the user on each web page. This significantly limits the flexibility of using Mathematics and requires the creation of a new teaching methodology. Technology as a set of techniques and methods for creating interactive electronic tutorials for WebMathematics is described in [6]. Let us briefly describe how the Continuity of Functions section is taught using Webmathematics. Interactive electronic textbook "Continuity and discontinuity points of functions", along with textbooks on other sections of Calculus and Linear Algebra, is posted on the website http://wm.iedu.ru, created at MIEM HSE. The title page of the manual is shown in Fig.3.

It follows from the figure that the manual, along with the theoretical part, contains Problem Solving section. The latter has two subsections: Tasks and Sample Execution work. In the sample, all calculations are performed using the Graph HTML form functions (Fig.4). In the input fields of this form, the data of the sample task are pre-entered, namely, formula defining the function under study, formula for elements sequence and the number of elements in the sequence. User First performs calculations of the sample problem, and then, having understood the method of solving problems of this kind, enters by editing the data of the problem he is solving and performs calculations.

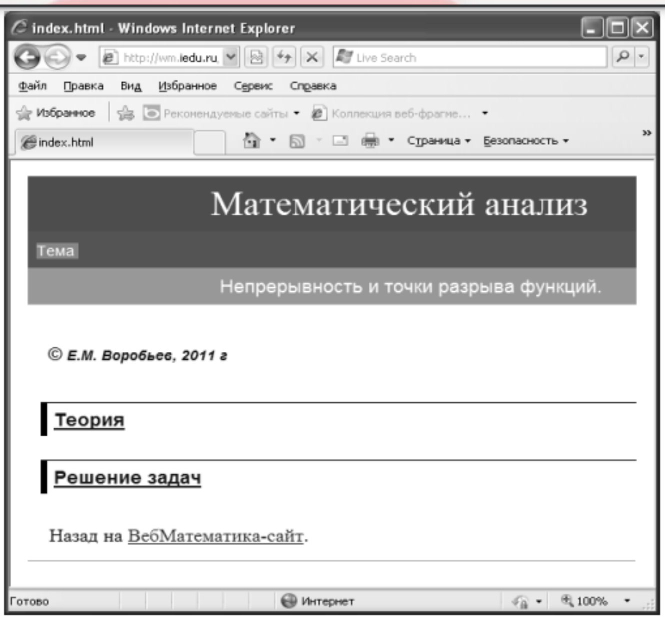

Рис.3. Титульная страница пособия «Непрерывность и точки разрыва функций»

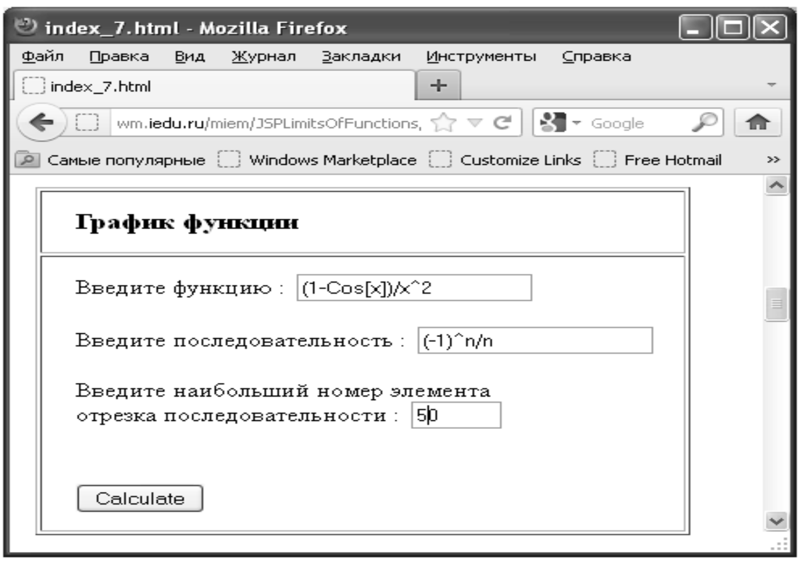

Рис.4. HTML-форма «График функции»

Conclusion

We have described the method of using computer algebras for educational purposes for teaching the discipline Mathematical Analysis. On the example of the Mathematics and Webmathematics systems, it is demonstrated that their use can significantly increase both the logical harmony and the accessibility of such a fundamental topic of analysis as continuity. The experience of on-line teaching students of MIEM NRU HSE specialty 230100 "Computer Science and Engineering" shows that students have absolutely no difficulties with interactive electronic textbooks.

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