BOSHLANG‘ICH FUNKSIYA VA ANIQMAS INTEGRALLARNI MAPLE DASTURIDA HISOBLASH

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Annatatsiya:
Ushbu ish bir o‘zgaruvchili funksiyani integral hisobiga bag‘ishlangan bo‘lib, unda integrallash masalalarini analitik va MAPLE dasturi yordamida yechish usullari berilgan.

Kalit so‘zlar: Funksiya, integral, boshlang‘ich funksiya, maple**.**

1-ta’rif. Agar D sohada f(x) funksiya aniqlangan bo‘lib, bu sohada hosilasi f(x) ga teng bo‘lgan F(x) funksiya mavjud bo‘lsa, u f(x) funksiyaning boshlang‘ich funksiyasi deyiladi.

Demak, ta’rif bo‘yicha F'(x)=f(x) bo‘lsa, F(x) funksiya f(x) ning boshlang‘ich funksiyasidir.

Masalan, f(x)=x uchun $F(x)=\frac{x^2}{2}$ boshlang‘ich funksiyadir, chunki,

$$F'(x)=\left(\frac{x^2}{2}\right)'=\frac{1}{2}\cdot 2x=x=f(x).$$

Shu misolda $F_1(x)=\frac{x^2}{2}+C$, funksiyani qarasak (buda C-qandaydir o‘zgarmas son)

$$F_1'(x)=\left(\frac{x^2}{2}+C\right)'=\left(\frac{x^2}{2}\right)'+C'=x+0=f(x)$$

bo‘lib, u ham f(x) ning boshlang‘ich funksiyasi bo‘lar ekan. $C\in R$ ekanligidan bu berilgan funksiyaning boshlang‘ich funksiyalarining cheksiz ko‘pligi va ular C- o‘zgarmas songa farq qilishi kelib chiqadi.

Berilgan f(x)=x funksiya va uning $F(x)=\frac{x^2}{2}$ boshlang‘ich funksiyasining grafigini qurish dasturi:

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> restart;
> with(Student[Calculus1]):
> AntiderivativePlot( x, x=-2..2, showclass, thickness=2);
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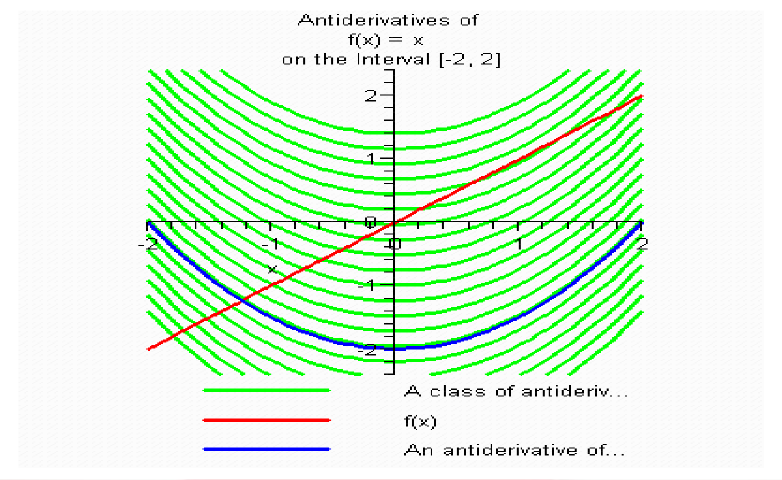
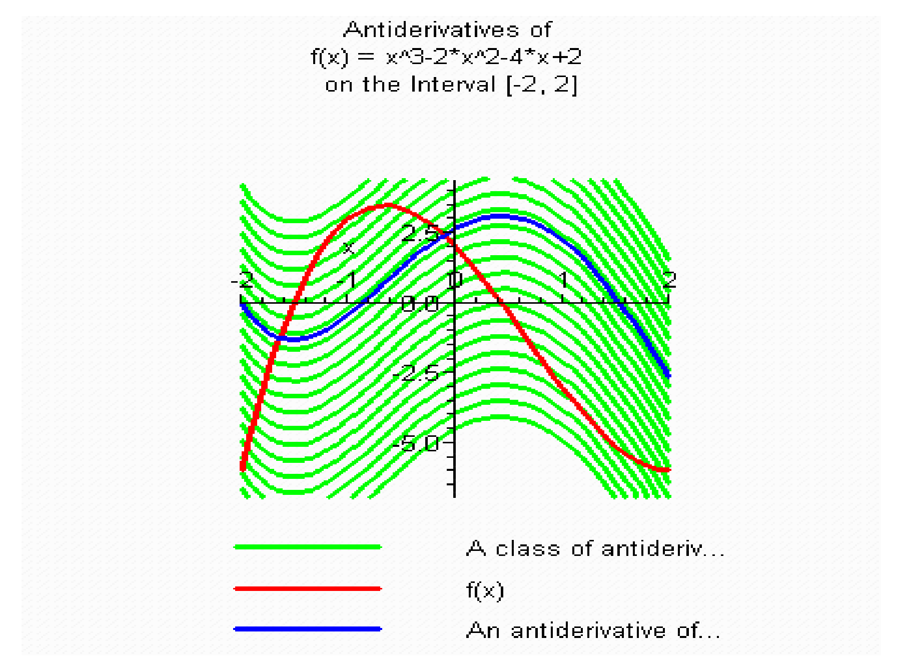


Masalan, f(x)=x^3-2x^2-4x+2 uchun $F(x)=\frac{x^4}{4}-\frac{2x^3}{3}-2x^2+2x+C$ boshlang‘ich funksiyadir. Bu funksiyalarning grafiklarini quramiz.

> restart;

> with(Student[Calculus1]):

> AntiderivativePlot(x^3-2*x^2-4*x+2, x=-2..2, showclass, thickness=2);



2-ta’rif. Agar D sohada aniqlangan f(x) funksiyaning boshlang‘ich funksiyasi F(x) mavjud bo‘lsa, boshlang‘ich funksiyalarining {F(x)+C :C∈ R} to‘plamini f(x) funksiyaning aniqmas integrali deyiladi va $\int f(x)dx$ kabi belgilanadi.

Bu yerda $\int$ -integral belgisi , f(x) –integral osti funksiyasi, f(x)dx –integral osti ifodasi, x– integral o‘zgaruvchisi deb yuritiladi.

Ta’rifga asoslangan holda $\int f(x)dx=F(x)+C$ ko‘rinishda yozish qabul qilingan bo‘lib, bu yerda F’(x)=f(x), C∈ R ixtiyoriy o‘zgarmasdir.

Masalan, yuqorida ko‘rilgan misollardan:

$\int x dx = \frac{x^2}{2} + C$,

$\int (x^3 - 2x^2 - 4x + 2)dx = \frac{x^4}{4} - \frac{2x^3}{3} - 2x^2 + 2x + C$

kabi yozish mumkin.

Berilgan funksiyaning boshlang‘ich funksiyasini, ya’ni aniqmas integralini topish jarayoni uni integrallash deb yuritiladi. Yuqoridagi ta’riflardan ko‘rinadiki, integrallash differensiallashga teskari amaldir. Buni hisobga olsak, differensiallashning asosiy qoidalaridan integrallash uchun quyidagi asosiy xossalar kelib chiqadi:

1^0. $\left[\int f(x)dx\right]' = f(x)$, $\quad d\left[\int f(x)dx\right] = f(x)dx$;

2^0. $\int du(x) = u(x) + C$, $\quad$ C – ixtiyoriy o‘zgarmas;

3^0. $\int Af(x)dx = A\int f(x)dx$, $\quad$ A – o‘zgarmas ko‘paytuvchi;

4^0. $\int \left[\sum_{i=1}^{n} A_i f_i(x)dx\right] = \sum_{i=1}^{n} A_i \int f_i(x)dx$, $A_i (i = \overline{1,n})$ - o‘zgarmaslar;

5^0. $\int f(x)dx = F(x) + C$ bo‘lib, x biror D sohada o‘zgarganda u(x)–differensiallanuvchi funksiya hamda uning qiymati $f(x)$ ning aniqlaninsh sohasiga tegishli bo‘lsa,

$$\int f(u(x))du(x) = F(u(x)) + C$$

bo‘ladi. Bu yerda du(x)=u’(x)dx ekanligini eslash lozimdir. Xususiy holda:

$u(x) = ax + b$ bo‘lganda $\int f(ax+b)dx = \frac{1}{a}F(ax+b) + C$ bo‘ladi

Integrallashning yuqoridagi xossalari asosida olinadigan quyidagi integrallar jadvalini keltiramiz. Ularni har birini Maple dasturida hisoblash buyruqlarini yozilishini ko‘rsatamiz.

Integral va uni Maple dasturida hisoblash buyruqarining jadvali

1. $\int x^\alpha dx = \frac{x^{\alpha+1}}{\alpha+1} + Ń$, $\quad -1 \neq \alpha \in R$, bo'lsa $\int dx = x + Ń$

> Int(x^n, x)=int(x^n, x);

2. $\int \frac{dx}{x} = \ln|x| + C$

> Int(1/x, x)=int(1/x, x);

3. $\int a^x dx = \frac{a^x}{\ln a} + C$, $\quad 1 \neq a \in R^+$; $\quad \int e^x dx = e^x + C$

> Int(a^x, x)=int(a^x, x);
> Int(exp(x), x)=int(exp(x), x);

4. $\int \sin x dx = -\cos x + C$

> Int(sin(x), x)=int(sin(x), x);

5. $\int \cos x dx = \sin x + C$

> Int(cos(x), x)=int(cos(x), x);

6. $\int tgx dx = -\ln|\cos x| + C$

> Int(tan(x), x)=int(tan(x), x);

7. $\int ctgx dx = \ln|\sin x| + C$

> Int(cot(x), x)=int(cot(x), x);

8. $\int \frac{dx}{\cos^2 x} = tgx + C$

> Int(1/cos(x)^2, x)=int(1/cos(x)^2, x);

9. $\int \frac{dx}{\sin^2 x} = -ctgx + C$

> Int(1/sin(x)^2, x)=int(1/sin(x)^2, x);

10. $\int \frac{dx}{\sqrt{1-x^2}} = \begin{cases} \arcsin x + C = arctg\frac{x}{\sqrt{1-x^2}} + C \\ -\arccos x + C = -arctg\frac{\sqrt{1-x^2}}{x} + C \end{cases}$

11. $\int \frac{dx}{\sqrt{a^2-x^2}} = \arcsin\frac{x}{a} + C \quad (= -\arccos\frac{x}{a} + C), \quad 0 \neq a \in R$

> Int(1/sqrt(a^2-x^2), x)=int(1/sqrt(a^2-x^2), x);

12. $\int \frac{dx}{1+x^2} = arctgx + C = -arcctgx + C$

13. $\int \frac{dx}{a^2+x^2} = \frac{1}{a} arctg\frac{x}{a} + C = -\frac{1}{a} arcctg\frac{x}{a} + C, \quad 0 \neq a \in R$

> Int(1/(a^2+x^2), x)=int(1/(a^2+x^2), x);

14. $\int \frac{dx}{x^2-a^2} = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C = -\frac{1}{2a}\ln\left|\frac{x+a}{x-a}\right| + C, \quad 0 \neq a \in R$

> Int(1/(a^2-x^2), x)=int(1/(a^2-x^2), x);

15. $\int \frac{dx}{\sqrt{x^2 \pm a^2}} = \ln\left|x + \sqrt{x^2 \pm a^2}\right| + C, \quad 0 \neq a \in R$

> Int(1/sqrt(x^2+a^2), x)=int(1/sqrt(x^2+a^2),x);
> Int(1/sqrt(x^2-a^2), x)=int(1/sqrt(x^2-a^2),x);

16. $\int \frac{dx}{\sin x} = \ln\left|tg\frac{x}{2}\right| + C = \ln\left|\frac{1}{\sin x} - ctgx\right| + C,$

> Int(1/sin(x),x)=int(1/sin(x),x);

17. $\int \frac{dx}{\cos x} = \ln\left|tg\left(\frac{x}{2} + \frac{\pi}{4}\right)\right| + C = \ln\left|\frac{1}{\cos x} + tgx\right| + C$

> Int(1/cos(x),x)=int(1/cos(x),x);

18. $\int \frac{f'(x)}{f(x)} dx = \ln|f(x)| + C$

> Int(diff(f(x),x)/f(x),x)=int(diff(f(x),x)/f(x),x);

19. $\int \frac{f'(x)}{\sqrt{f(x)}} dx = 2\sqrt{f(x)} + C$

> Int(diff(f(x),x)/sqrt(f(x)),x)= int(diff(f(x),x)/sqrt(f(x)),x);

Umuman olganda maple dasturi yordamida matematik amallarni bajarish ham tushunarli, ham ko‘rgazmalilikni oshirgan holda talabalarning ushbu fanni o‘zlashtirish ko‘rsatkichlarini oshiradi.

Foydalanilgan adabiyotlar ro‘yxati

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