ANALYSIS OF MATHEMATICAL MODELS OF UNSTABLE WATER MOVEMENT IN WATER ECONOMY FACILITIES TAKING INTO ACCOUNT THE MULTI-DIMENSIONAL DISTRIBUTION OF PARAMETERS IN SPACE

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ANNOTATION

The article describes the analysis of the mathematical models of unstable water movement in water management facilities, taking into account the multidimensional distribution of parameters in space.

Key words and phrases: space, multidimensional, water management objects, unsteady water movement, mathematical model.

Currently, approximate methods for solving one-dimensional equations of unsteady water motion are very common and are widely used in practical calculations. Two directions should be emphasized here: the use of improved equations and the use of complete systems of Saint-Venant equations.

In the one-dimensional case, the Saint-Venant equation has the following form:

$$B\frac{\partial z}{\partial t}+\frac{\partial Q}{\partial x}=q$$

$$\frac{1}{g\omega}\left(\frac{\partial Q}{\partial t}+2v\frac{\partial Q}{\partial x}\right)+\left[1-\left(\frac{v}{c}\right)^2\right]\frac{\partial z}{\partial x}=\left[i+\frac{1}{B}\left(\frac{\partial \omega}{\partial x}\right)_{h=const}\right]\left(\frac{v}{c}\right)^2-\frac{Q|Q|}{K^2}, \qquad (1)$$

here: $Q=Q(x\ ,t)$ – water consumption; z=z(x, t) is the ordinate of the free surface; г – gravitational constant; i – lower slope; B=B(z) is the flow width along the surface of the live section; ω=ω(z) is the surface of the live section of the stream; c=c(z) is the propagation speed of small waves; K=K(z) is the consumption module.

An important advantage of hydraulic models is their versatility. They are used both in the use and design of sections of rivers and canals. Disadvantages of hydraulic models are mainly related to processes in riverbeds, where non-transit zones - thickets or other places of the river where water hardly moves - appear. Non-transit zones act as reservoirs, so such zones should not be taken into account in the live section of the stream. Methods for separating transit zones have not yet been developed, as a result of which they are not taken into account in common one-dimensional equations of water movement. In well-maintained canals, the formation of no-transit zones is almost never observed, and as a result, these disadvantages of hydraulic models are insignificant.

Thus, the greatest interest for the study of dynamic processes in water management objects and systems is manifested in hydraulic models.

The given models can be classified according to the solution methods used. Existing methods of solving Saint-Venant equations are conditionally divided into three groups. The first group includes the solutions obtained as a result of attempts to find the general integral of the Saint-Venant equations with the help of strict mathematical analysis, when the method of differential properties is applied with the use of finite difference equations. The second group consists of solutions found using mathematical analysis, including the theory of small-amplitude waves. The third group includes the solutions obtained as a result of approximate integration of Saint-Venant equations with preliminary substitution of finite difference equations.

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Models based on solving modified one-dimensional Saint-Venant equations [1]. The convection-diffusion model is based on neglecting the inertial terms of the equations and has the following form:

$$\frac{\partial Q}{\partial t}+\left(\frac{Q}{K}\frac{\partial K}{\partial h}\right)\frac{\partial Q}{\partial x}-\frac{K^2}{2b|Q|}\frac{\partial^2 Q}{\partial x^2}=0, \qquad (2)$$

Here K is the consumption module.

When the slope of the free surface is ignored, we get the kinematic wave equation:

$$\frac{\partial\omega}{\partial t}+\frac{\partial Q}{\partial x}=0 \qquad (3)$$
$$Q=\omega c\sqrt{Ri}$$

Small-amplitude wave theory models [2] show that all changes in hydraulic elements caused by wave motion are mostly small quantities, so the squares of these quantities and their products can be neglected. By linearizing the Saint-Venant equation of steady motion, its values are reduced to linear equations of hyperbolic type with constant coefficients determined for the initial singular mode.

The advantage of the above models is the use of a small number of generally accepted and repeatedly tested initial positions, which allows for a precise and rigorous mathematical formulation of the problems that arise.

In most cases, based on the hydrodynamic theory, it is possible to make detailed calculations of the progress of relevant physical phenomena in a multidimensional spatial domain and in time. An example of such a successful application of the theory is the movement of water in the form of long waves. Two-dimensional processes in riverbeds, lakes, canals and reservoirs are the most important in practice.

Multidimensional hydrodynamic processes characterized by long-wave disturbances find comparisons in mechanics and geophysics, acoustics, gas dynamics, hydraulics, meteorology, seismology and other fields of science.

The theory of long waves belongs to the classical branches of hydrodynamics. The initial state of the theory is the hydrostatic law for pressure:

$$p(x,y)=\rho g(\xi(x,y)-z(x,y))+p_0(x,y), \qquad (4)$$

where x, u are horizontal coordinates, the XOU coordinate plane corresponds to the fixed surface of the liquid, the vertical Z axis is directed upwards; ξ - liquid level above the equilibrium state, ρ - density of water, g - acceleration of gravity. Since ρ is constant everywhere, it allows us to exclude the consideration of internal waves.

Neglecting vertical acceleration leads to the law of hydrostatics. This process allows to reduce the size of the field of study and consider the movement in the two-dimensional XOU plane.

The most widely used applications, problems solved, and valuable results obtained using two-dimensional long wave theory are undoubtedly those in the study of astronomical ocean tides. The entire history of the development of the wave theory since Newton's time consists of a gradual complication of the mathematical description based on some immutable physical principles. The static theory of waves already includes the basic postulates of the theory of shallow water, and in this case the equations represent the balance of the pressure gradient force and the horizontal projections of the wave-forming force. The foundations of the theory and its mathematical description have not changed essentially in 200 years. However, the variety, nature and complexity of tasks and their solutions make even simple calculations difficult.

The first ideas related to the study of two-dimensional water flows using the apparatus of Newtonian mechanics belong to Newton himself, who developed the basic rules of static theory. In Eri's research, he described many issues about free and forced waves in idealized basins.

The generalization of multidimensional problems of long wave propagation leads to boundary value problems for hyperbolic equations if we do not consider the motion to be periodic with time. Solving hyperbolic equations, in general, seems to be more convenient, because in addition to the stable oscillation of a certain frequency, which appears under the influence of a periodic external force or boundary mode, this phenomenon, including between the initial and stable states allows describing transitions.

The simplest cases of motion lead to the wave equation, and there is a group of satisfactorily solved problems based on this simplified description. At the same time, most of the important non-linear problems and high requirements for the accuracy of the result are expressed in the form of mixed problems for quasi-linear hyperbolic systems. Such tasks are considered quite complex, and their appearance in this direction marks the beginning of the modern research stage.

Currently, the classical theory of long waves affects various aspects of the changes taking place. Hydrodynamic analysis of the process led to a general approach to the study of long-wave motions based on shallow water theory. Courant's [3] research on the behavior of quasi-linear hyperbolic equations made it possible to obtain the correct conditions for setting a mixed problem for systems of symmetric matrix equations, in particular, equations describing the movement of waves in shallow water. Advances in computing techniques and numerical methods for integrating differential equations have created the basis for extensive research on the creation of mathematical models of relevant geophysical phenomena. And finally, in the field of practical implementation of the theory of improving the methodology and technique of hydrometeorological observations, they made it possible to create hydrodynamic methods of calculating these phenomena in various real conditions[4]. We will consider some of these issues in detail.

The discontinuity equation for an incompressible fluid is expressed as:

$$divV=0, \qquad (5)$$

where V is the velocity vector.

The use of the hydrodynamic theory of shallow water in solving various practical problems is related to expressing them as some boundary problems for a system of differential equations. In this case, the morphometry of the water reservoir, the driving and distribution forces, the initial state of the process in the entire spatial domain should be shown in sufficient detail, and some methods of finding the solution of the equations should also be proposed[5]. The set of these elements describes the mathematical model of the phenomenon under study, and since the solution of the system of differential equations must be continuously dependent on the coefficients of the equations, the right-hand side, the initial and boundary conditions, the efficiency and accuracy of the model are many mathematical problems, it is clear that it is determined by the factors of hydrological and meteorological nature.

At the same time, due to the complexity of natural phenomena, finding functional tasks and analytical solutions of relevant factors often does not meet the requirements of practice. This applies, for example, to the elements of hydrotechnical design justification and to the construction of methods for predicting changes in the water level associated with various disturbances. It should be noted that compared to other oceanological issues, research on long-wave disturbances is successful. In fact, the creation of new methods for calculating hydrometeorological phenomena will be connected with the success of the hydrodynamic theory and the improvement of the methodology and techniques of observations[6]. A good example of such research is advances in numerical weather forecasting. It was here that the "hierarchy" and principles of processes in the atmosphere and the selection of the desired effects from the system of general equations were first expressed.

References:

1. Атавин А.А. Расчет неустановившегося течения воды в разветвленных системах речных русел и каналов // Динамика сплошной среды. Вып. – 22, Новосибирск, 1975, С.25-39.

2. Атавин А.А., Васильев О.Ф., Воеводин А.Ф., Шугрин С.М. Численные методы решения одномерных задач гидравлики // Водные ресурсы, 1983, № 4 С. 38–47.

3. Babuska I. Approximation by hill functions. Tech. Note BN-648.— Inst. for Fluid Dynam. Appl.'Math. Univ. of Maryland, 1970.

4. Вольцингер Н. Е., Пясковский Р. В. Теория мелкой воды. Океанологические задачи и численные методы. – Л.: Гидрометеоиздат. 1977. – 320 с.

5. Грушевский М.С. Неустановившееся движения воды в реках и каналах. – Л.: Гидрометеоиздат, 1982. – 289 с.

6. Zlamal M. On the finite element method. //Numer. Math 1968 12, № 5, p. 394—409.