BASIC THEOREMS OF DIFFERENTIAL CALCULUS AND THEIR APPLICATION

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Annotation. This article gives you some easy ways to solve common . Basic theorems of differential calculus and their application

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We can often use theorems on derivative functions to solve some problems. These theorems play an important role in checking functions.
Theorem 1 (Farm theorem).
$f(x)$ function $X \subset R$ given in the package. $x_0 \in X$ for the circumference of the point $U_\delta(x_0) = (x_0 - \delta,\ x_0 + \delta) \subset X \quad (\delta > 0)$ The following conditions must be met:
1) $\forall x \in U_\delta(x_0)$ da $f(x) \le f(x_0) \qquad (f(x) \ge f(x_0)),$
2) $f'(x_0)$
be available and limited.
Then $f'(x_0) = 0$ is being..
Let's say, $\forall x \in U_\delta(x_0)$ in $f(x) \le f(x_0)$ let it beObviously, in this case
$f(x) - f(x_0) \le 0$
will be.Conditionally $f(x)$ function x_0 limited in point $f'(x_0)$ yield.Then

$$f'(x_0) = \lim_{x\to x_0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x\to x_0+0} \frac{f(x) - f(x_0)}{x - x_0} = \lim_{x\to x_0-0} \frac{f(x) - f(x_0)}{x - x_0}$$

Will be.At the moment , $x > x_0$ will be

$$\frac{f(x) - f(x_0)}{x - x_0} \le 0 \quad \Rightarrow \quad \lim_{x\to x_0+0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0) \le 0,$$

$x < x_0$ will be

$$\frac{f(x) - f(x_0)}{x - x_0} \ge 0 \quad \Rightarrow \quad \lim_{x\to x_0-0} \frac{f(x) - f(x_0)}{x - x_0} = f'(x_0) \ge 0$$ from $f'(x_0) = 0$ It turns out that.

Theorem 2 (Roll theorem).Suppose, $f(x)$ function $[a,\ b]$ to meet the following conditions:
1) $f(x) \in C[a,\ b],$
2) $\forall x \in (a,\ b)$ in $f'(x)$ available and limited,

3) $f(a)=f(b)$ let it be. $x_0 \in (a,\ b)$ $f'(x_0)=0$

Conditionally $f(x) \in C[a,\ b]$. According to Weierstrass's second theorem $f(x)$ function $[a,\ b]$ at its maximum and minimum values, $c_1,\ c_2$ points $(c_1,\ c_2 \in [a,\ b])$ found'

$f(c_1)=\max\{f(x) \mid x \in [a,\ b]\}$,

$f(c_2)=\min\{f(x) \mid x \in [a,\ b]\}$

Has been.

If $f(c_1)=f(c_2)$ been, Then $[a,\ b]$ in $f(x)=const$ is being, $\forall x_0 \in (a,\ b)$ at $f'(x_0)=0$.

If $f(c_1)>f(c_2)$ be, thats $f(a)=f(b)$ because $f(x)$ function $f(c_1)$ and $f(c_2)$ to at least one of the values $[a,\ b]$ the interior of the segment x_0 $(a<x_0<b)$ .reach the point According to the farm theorem $f'(x_0)=0$ will be. ►

3-theorem (Lagranj by theorem).Suppose, $f(x)$ function $[a,\ b]$ at will be,fulfill the following conditions:

$f(x) \in C[a,\ b]$,

$\forall x \in (a,\ b)$ at $f'(x)$ the product is available and limitedIn that case it is so $c \in (a,\ b)$ poind found ,

$f(b)-f(a)=f'(c)(b-a)$

will be.

This $F(x)=f(x)-f(a)-\dfrac{f(b)-f(a)}{b-a}(x-a)$ (1)

Let's look at the function. This function satisfies all the conditions of the Roll theorem. At the same time, its a product

$$F'(x)=f'(x)-\frac{f(b)-f(a)}{b-a}$$

will be.According to the roll theorem, so c $(c \in (a,\ b))$ the point is found,

$F'(c)=0$ (2)

Will be.

(1) and (2) from equations

$f'(c)-\dfrac{f(b)-f(a)}{b-a}=0$,that is $f(b)-f(a)=f'(c)(b-a)$

will occur.

1-result. Let's say, $f(x)$ function $(a,\ b)$ at $f'(x)$,having a product $\forall x \in (a,\ b)$ at $f'(x)=0$ being to. Then $\forall x \in (a,\ b)$ at $f(x)=const$ will be.

$x,\ x_0 \in (a,\ b)$ take, edges x and x_0 in the segment $f(x)$ using Lagrange's theorem on the function $f(x)=f(x_0)=const$ being found. ►

2-result. $f(x)$ and $g(x)$ function $(a,\ b)$ at $f'(x)$, $g'(x)$,products $\forall x \in (a,\ b)$ in $f'(x)=g'(x)$ been. Then $\forall x \in (a,\ b)$ in $f(x)=g(x)+const$ will be.

This is proof of the result $f(x)-g(x)$ by applying result 1 to the function.Theorem 4 (Cauchy Theorem). Let, and let the functions fulfill the following conditions.

1) $f(x)\in C[a,\ b],\ g(x)\in C[a,\ b]$,

2) $\forall x\in(a,\ b)$ da $f'(x)$ va $g'(x)$ crops are available and limited;

3) $\forall x\in(a,\ b)$ da $g'(x)\neq 0$ will be.

Then $c\in(a,\ b)$,the point is found

$$\frac{f(b)-f(a)}{g(b)-g(a)}=\frac{f'(c)}{g'(c)}$$

Willbe.

First of all $g(b)\neq g(a)$ We emphasize that because $g(b)=g(a)$ if so, then according to Roll's theorem $c\in(a,\ b)$ the point would be found $g'(c)=0$ would be This is contrary to condition

3) The following

$$\Phi(x)=f(x)-f(a)-\frac{f(b)-f(a)}{g(b)-g(a)}[g(x)-g(a)]\quad (x\in[a,\ b])$$

$c\in(a,\ b)$ found point ,

$\Phi'(c)=0$ will be (3)

$$\Phi'(x)=f'(x)-\frac{f(b)-f(a)}{g(b)-g(a)}g'(x)\ (4)$$

Obviouly,

(3) and (4) relationship

$$f'(c)-\frac{f(b)-f(a)}{g(b)-g(a)}g'(c)=0$$

that is $\dfrac{f(b)-f(a)}{g(b)-g(a)}=\dfrac{f'(c)}{g'(c)}$

will occur.

Example 1. $\forall x',\ x''\in R$ for $|\sin x'-\sin x''|\leq|x'-x''|$ prove the inequality.

Let's say, $x'<x''$ will be. $f(x)=\sin x$ in $[x',\ x'']$ We apply Lagrange's theorem.That's it $c\in(x',\ x'')$ the point is that,

$|\sin x'-\sin x''|=|\cos c|\cdot(x''-x')$

willbe.If $\forall t\in R$ at $|\cos t|\leq 1$ Given that, then from the above relationship,

$|\sin x'-\sin x''|\leq|x'-x''|\qquad(\forall x',\ x''\in R)$

Being.

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