

KARRALI INTEGRALLARNI HISOBLASHNING GEOMETRIK USULI

Shamsiyeva Nodira Bahodir qizi

O'zMU Jizzax filiali, Amaliy matematika fakulteti talabasi

Sharipova Sadoqat Fazliddinovna

ilmiy rahbar, O'zMU Jizzax filiali katta o'qituvchisi

ANNOTASIYA

Karrali integrallarni hisoblash masalalari uchun sohani aniqlash juda muhimdir. Geometrik usul – bu sohani aniqlashning ko'p qo'llaniladigan usullaridan biridir.

ABSTRACT

Determination of the region for problems of calculating multiple integral is very important. Geometric method is one of the methods determining the region usually used.

АННОТАЦИЯ

Определение области для задач вычисления кратного интеграла очень важно. Геометрический метод – один из часто используемых методов определения этой области.

Kalit so'zlar: soha, karrali integral, geometrik usul, funksiyalar grafigi.

Ushbu ishda karrali integral va uch karra integralni o'rganilgan. Masalada ko'rsatilgan sohaga bog'liq holda biz takroriy usul yoki karrali integral uchun qutb koordinatalar sistemasida yangi o'zgaruvchi va uch karrali integralda silindrik, sferik koordinatalar sistemasidagi o'zgaruvchi yordamida yangi sohani kiritishimiz mumkin. Karrali integrallarni hisoblashning geometrik usulidan foydalanish uchun sohani ifodalovchi funksiyaning grafigini chizamiz. Grafikdan takroriy usuldan foydalanish uchun chap, o'ng, yuqori va pastki chegaralarni yoki qutb koordinatali o'zgaruvchilar usulini qo'llash uchun φ burchakni yoki sferik koordinatalar yordamida o'zgaruvchilarni almashtirish usulidan foydalanish uchun φ, θ burchaklarni aniqlash mumkin. Haqiqatan funksiya grafigini ko'rib chiqaylik. Karrali integralni hisoblashning geometrik usulidan foydalanishni bir nechta misollarda ko'rib chiqaylik.

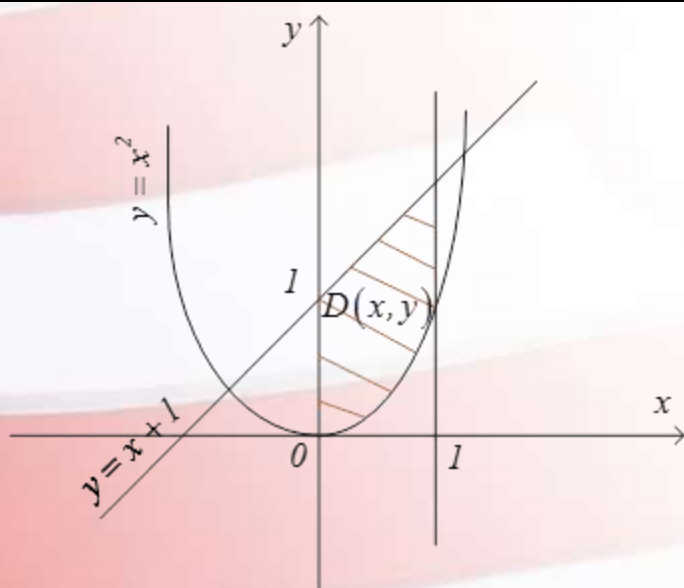
1. Karrali integralni hisoblang.

$$\iint_{D(x,y)} (xy - y^2) dx dy, \quad D(x,y) = \{(x,y) / x = 0, x = 1, y = x^2, y = x + 1\}$$

Yechish: Avval sohani aniqlovchi funksiyalarining grafigini chizamiz

$$D(x,y): x = 0, x = 1, y = x^2, y = x + 1$$

Biz $D(x,y)$ sohani 1-rasmdagi kabi hosil qilamiz:

**1-rasm. Soha**

$D(x, y)$ soha grafigidan quyidagini hosil qilamiz:

$$D(x, y) = \{(x, y) / 0 \leq x \leq 1, x^2 \leq y \leq x + 1\}$$

Bundan

$$I = \iint_{D(x, y)} (xy - y^2) dx dy = \int_0^1 dx \int_{x^2}^{x+1} (xy - y^2) dy = \int_0^1 \left[\left(x \frac{y^2}{2} - \frac{y^3}{3} \right) \Big|_{x^2}^{x+1} \right] dx = -\frac{97}{168}.$$

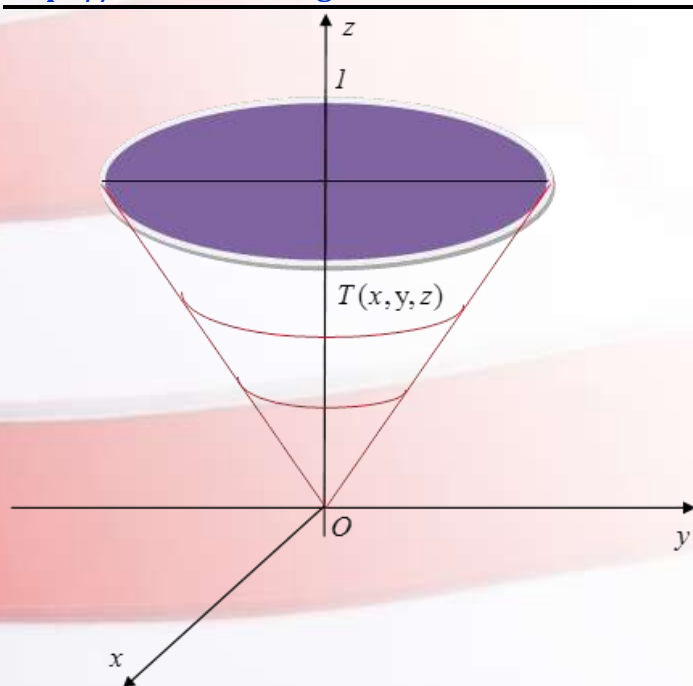
2. Uch karrali integralni hisoblaymiz:

$$I = \iiint_{T(x, y, z)} \sqrt{x^2 + y^2} dx dy dz, T(x, y, z) = \{(x, y, z) / z = \sqrt{x^2 + y^2}, z = 1\}$$

Yechish: Avval

$$T(x, y, z); z = \sqrt{x^2 + y^2}, z = 1$$

sohani aniqlovchi funksiyaning grafigini chizamiz (2-rasm):



2-rasm

Silindrsimon koordinatalar yordamida o'zgaruvchilarni almashtirish usulidan foydalanamiz:

$$\begin{cases} x = r \cos \varphi \\ y = r \sin \varphi, (x, y) \in D(x, y), (x, y, z) \in T(x, y, z) \\ z = z \end{cases}$$

bu yerda $D(x, y) - T(x, y, z)$ sohaning Oxy tekisligidagi proyeksiyasi.

$D(x, y) = \{(x, y) / x^2 + y^2 \leq 1\}$ sohadan

$$D(\varphi, r) = \{(\varphi, r) / 0 \leq \varphi \leq 2\pi, 0 \leq r \leq 1\}$$

Haqiqatan ham, $T(x, y, z) = \{(x, y, z) / (x, y) \in D(x, y), \sqrt{x^2 + y^2} \leq z \leq 1\}$

Shunday qilib, integralni hisoblash sohasi quyidagi shaklga ega:

$$\begin{aligned} T(\varphi, r, z) &= \{(\varphi, r, z) / (\varphi, r) \in D(\varphi, r), 0 \leq r \leq 1\} \\ &= \left\{ \frac{(\varphi, r, z)}{0} \leq \varphi \leq 2\pi, 0 \leq r \leq 1, r \leq z \leq 1 \right\} \end{aligned}$$

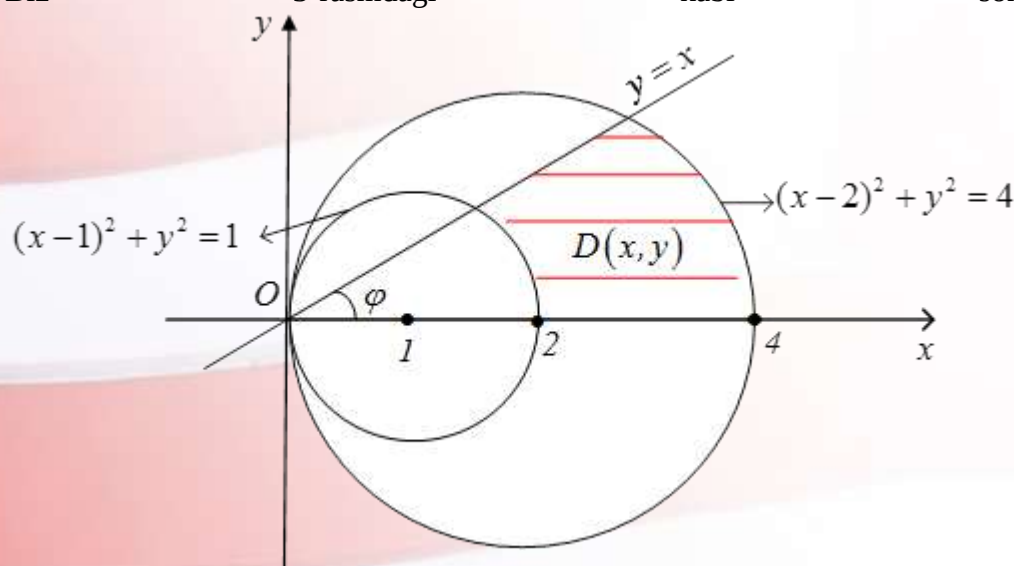
$$\rightarrow I = \iiint_{T(x, y, z)} \sqrt{x^2 + y^2} dx dy dz = \iiint_{T(\varphi, r, z)} r \cdot r d\varphi dr dz = \int_0^{2\pi} r \cdot d\varphi \int_0^1 r^2 dr \int_r^1 dz = \frac{\pi}{6}$$

$D(x, y) = \{(x, y) / (x-1)^2 + y^2 \geq 1, (x-2)^2 + y^2 \leq 4, y \leq x, y \geq 0\}$ sohani isbotlang.

Yechish: Avval

$$D(x, y), x = 0, x = 1, y = x^2, y = x + 1$$

sohada berilgan funksiyalarining grafisini chizamiz.



Biz qutbli koordinatalar yordamida o'zgaruvchilarni almashtirish usulidan foydalanamiz:

$$\begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \end{cases}$$

3-rasmdan ko'rinib turibdiki:

$$0 \leq \varphi \leq \frac{\pi}{4}.$$

$$\begin{cases} x = r \cos \varphi \\ y = r \sin \varphi \end{cases} \text{ ifodani } \begin{cases} (x-1)^2 + y^2 \geq 1 \\ (x-2)^2 + y^2 \leq 4 \end{cases} \text{ tengsizliklarga qo'yib} \\ 2 \cos \varphi \leq r \leq 4 \cos \varphi$$

tengsizlikni hosil qilamiz.

Darhaqiqat, $D(x,y)$ sohaning yuzi:

$$S = \iint_{D(x,y)} dx dy = \int_0^{\frac{\pi}{4}} \int_{2 \cos \varphi}^{4 \cos \varphi} r dr d\varphi = \int_0^{\frac{\pi}{4}} 6 \cos^2 \varphi d\varphi = \frac{3(2 + \pi)}{4}$$

Foydalanilgan adabiyotlar:

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